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Non-topological soliton in three-dimensional supergravity

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Abstract

We construct numerical solutions for non-topological solitons in three-dimensional $U(1)$-gauged $\mathcal{N} = 2$ supergravity. We find the region of the solutions showing with the BTZ mass, the angular momentum and the magnetic flux and discuss the relation among the physical parameters for various values of a cosmological constant.

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I. INTRODUCTION

As is well known, three-dimensional pure Einstein gravity is a topological theory and has no propagating degrees of freedom. The solutions are locally flat apart from conical singularities at the location of matter sources in no cosmological constant case [1,2]. Moreover, cosmological gravity and supergravity are solvable in many cases [3–9].

Three-dimensional supergravity has been investigated in various situations intensively. As particular examples, the non-linear sigma models coupled to $\mathcal{N}$-extended supergravity were studied [10]. The geometry of the target manifolds parametrized by scalar fields were classified completely: the target space is Riemanian, Kähler, and quaternionic for $\mathcal{N} = 1, 2$ and $\mathcal{N} = 3$ respectively. For $\mathcal{N} = 4$ it generally decomposes into two separate quaternionic spaces, associated with inequivalent supermultiplets. For $\mathcal{N} = 5, 6, 8$ there is a symmetric space for any given number of supermultiplets. For $\mathcal{N} = 9, 10, 12$ and 16 there are only theories based on a single supermultiplet associated with coset spaces with the exceptional isometry groups F_4, E_6, E_7 and E_8 respectively.

Moreover, the three-dimensional maximal ($\mathcal{N} = 16$) gauged supergravity were constructed [11,12]. The duality relation between the gauge fields and the scalar field plays an important role in the derivation of the ungauged $\mathcal{N} = 16$ supergravity: in order to expose its global E_8 isometry, all vector fields obtained by dimensional reduction of the $D = 11$ supergravity in an eight-torus must be dualized into the scalar fields. Moreover, in order for the supergravity to gauge, the Chern-Simons term (rather than the Yang-Mills term) must be required in the Lagrangian.

Recently the supersymmetric vortex solutions in $U(1)$-gauged $D = 3, \mathcal{N} = 2$ supergravity whose scalar sector is an arbitrary Kähler manifold with $U(1)$ isometry were constructed [13]. It has been found that the Einstein equations and the matter field equations of the model can be recast into a set of self-duality equations for a specific eighth-order choice of the Higgs potential which reduces to the sixth-order potential of the flat space model when the Newton gravitational coupling constant is set to zero. The Abelian Chern-Simons Higgs model in

three-dimensional Minkowski space-times and its vortex solutions were studied by [14–16]. It was shown that the model with a specific sixth-order Higgs potential admits topologically stable vortex solutions and non-topological soliton solutions with nonzero flux and charge, which satisfy the Bogomol'nyi-type or the first order self-duality equations [17]. In the Bogomol'nyi limit, there is a (first-order) phase transition point between the symmetric (non-topological solutions) and the asymmetric phase (topological vortex solutions).

In this paper, we construct the non-topological solitons, which do not preserve the supersymmetry, in three-dimensional $U(1)$-gauged $\mathcal{N}=2$ supergravity and find the region where the solutions can exist for various value of a cosmological constant. A further motivation for studying the non-topological solitons is that this model provided us with the simplest model of relativistic stars as self-gravitating systems. Rotating boson stars with large self-coupling constant in three dimensions were studied in [18], for example. The study of the non-topological soliton will lead to a new aspect of the self-gravitating systems.

The organization of this paper is as follows; in the next section we introduce the model and the basic ingredients. For the construction of the $U(1)$-gauged $D=3$, $\mathcal{N}=2$ supergarvity Lagrangian, we refer the reader to the existing literature [10–13]. In section III, we restrict ourselves to the simple case of the Kähler manifold and a circularly symmetric ansatz for the metric which approaches asymptotically the BTZ configuration [19], which is asymptotically anti-de Sitter space (rather than asymptotically flat) and does not preserve the supersymmetry in general. In this setting we obtain numerically the non-topological solitons with zero-winding number case for various values of a cosmological constant. In the section IV, we show the numerical results and discuss the relation among the physical parameters for various values of a cosmological constant.

II. $U(1)$-GAUGED $D=3$, $\mathcal{N}=2$ SUPERGARVITY

In this section we consider ungauged $\mathcal{N}=2$ three-dimensional supergarvity. Assuming a $U(1)$ isometry of the Kähler potential, we apply the Noether procedure to obtain the

$U(1)$-gauged $\mathcal{N}=2$ supergarvity [10,13]. The field content of the ungauged theory is the following.

- The $\mathcal{N}=2$ supergravity multiplet

$$\{e_\mu^a, \psi_\mu\} \tag{1}$$

  contains a graviton e_μ^a and two gravitini which are assembled into one complex spinor ψ_μ.

- The $\mathcal{N}=2$ scalar multiplet

$$\{\phi^\alpha, \lambda^\alpha\} \tag{2}$$

  contains the fermions $(\lambda^\alpha, \bar{\lambda}^{\bar{\alpha}})$ and scalars $(\phi^\alpha, \bar{\phi}^{\bar{\alpha}})$ with $\alpha = 1, \ldots, p$, respectively in the complex notation. The matter sector is obtained by p copies of the $\mathcal{N}=2$ scalar multiplet. The scalar fields define a Kähler manifold of real dimension $2p$, characterized by its Kähler potential $K(\phi^\alpha, \bar{\phi}^{\bar{\alpha}})$.

It is assumed that the ungauged $\mathcal{N}=2$ supergarvity is invariant under the global $U(1)$ isometry. As in the maximally gauged supergravity theories [11,12], the Chern-Simons term is required, in order to gauge the $U(1)$ Kähler isometry. Note that this term is topological and hence does not introduce new propagating degrees of freedom in the gauged theory.

Moreover, the g-dependent terms, due to the Chern-Simons terms, give rise to extra terms in supersymmetry transformations. In order to compensate these terms, the extra Yukawa-type bilinear fermionic terms and a scalar potential must be added to the Lagrangian.

Then the bosonic parts of the $U(1)$-gauged $D=3$, $\mathcal{N}=2$ supergarvity Lagrangian is given by:

$$\mathcal{L} = \frac{1}{4}eR - eG_{\alpha\bar{\alpha}}(\phi,\bar{\phi})\mathcal{D}_\mu\phi\mathcal{D}^\mu\bar{\phi} - \frac{1}{8}g\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} - eg^2V, \tag{3}$$

where $D_\mu\phi \equiv (\partial_\mu + igA_\mu)\phi$ is a covariant derivative and $G_{\alpha\bar{\alpha}}(\phi,\bar{\phi})$ denotes the Kähler metric $G_{\alpha\bar{\alpha}}(\phi,\bar{\phi}) = \partial_\alpha\partial_{\bar{\alpha}}K(\phi,\bar{\phi})$. $F_{\mu\nu}$ is an Abelian field strength and g is a gauge coupling constant. $V = V(\phi,\bar{\phi})$ is a real scalar potential, which will be mentioned in the next section.

III. NON-TOPOLOGICAL SOLITON

We restrict ourselves to the simple case of the Lagrangian (3) considered in the preceding section. We adopt a single complex scalar field case, that is $p = 1$. Moreover we assume that the K is a function of $R = |\phi|$ only and the Kähler potential is $K(R) = R^2$ in order to obtain the canonical kinetic term for the scalar field. We thus find that the $G \equiv G_{1\bar{1}}$ is unity.

Then, the eighth-order scalar potential is reduced to:

$$V = -2g^2[\phi^8 - 2(2c+1)\phi^6 + 2(3c^2 + 2c + b)\phi^4 - 2c(2c^2 + c + 2b)\phi^2 + c^4 + b^2 + 2bc^2], \quad (4)$$

where b and c are arbitrary real constant parameters.

We assume the three-dimensional metric for a circularly symmetric spacetimes as

$$ds^2 = -e^{-2\delta(r)}\Delta(r)dt^2 + \Delta^{-1}(r)dr^2 + r^2(d\theta^2 - \Omega(r)dt)^2, \quad (5)$$

where δ, Δ, and Ω are functions of the radial coordinate r only.

For the single complex scalar field, we also assume the following dependence on the coordinates:

$$\phi = R(r)e^{-in\theta}, \quad (6)$$

where $R(r)$ is a function of the radial coordinate r only and n is a constant. We impose the boundary condition that $\phi(r)$ approaches zero when the radial coordinate r is taken to infinity, in order to obtain the non-topological soliton.

For the vector field A_μ, we choose the gauge, in order to require finite energy solution for the scalar field at radial infinity, in which

$$A_r = 0, \quad A_\theta = P(r) + \frac{n}{g}, \quad A_t = W(r), \quad (7)$$

where $P(r)$ and $W(r)$ are functions of the radial coordinate r only.

Varying the Lagrangian with respect to the gauge field, the scalar field and the metric yields equations of motion. The equation of motion for the Chern-Simons term induces the first order equations

$$\epsilon^{\mu\nu\rho}F_{\nu\rho} = 8ire^{-\delta}(\bar{\phi}\mathcal{D}^{\mu}\phi - \phi\mathcal{D}^{\mu}\bar{\phi}), \tag{8}$$

and two of these equations take the form

$$\partial_r W = -4gr\Delta^{-1}e^{\delta}R^2\Omega(W+\Omega P) + 4gR^2Pr^{-1}e^{-\delta}, \tag{9}$$

$$\partial_r P = 4gr\Delta^{-1}R^2e^{\delta}(W+\Omega P), \tag{10}$$

and the third one ($\mu = r$) is automatically satisfied.

The scalar field equation is reduced to the form:

$$R'' + \left(\frac{\Delta'}{\Delta} + \frac{1}{r} - \delta'\right)R' + \frac{g^2e^{2\delta}R}{\Delta^2}(W+\Omega P)^2 - \frac{g^2P^2R}{r^2\Delta} = \frac{1}{2\Delta}\frac{\partial V}{\partial R}. \tag{11}$$

The Einstein equation is also reduced to:

$$\frac{1}{2}\frac{\Delta'}{r} + \frac{1}{4}r^2e^{2\delta}(\Omega')^2 = -\kappa^2\left[\frac{e^{2\delta}}{\Delta}g^2R^2(W+\Omega P)^2 + \Delta(R')^2 + \frac{(gPR)^2}{r^2} + V\right], \tag{12}$$

$$\frac{\Delta}{r}\delta' = -2\kappa^2\left[\frac{e^{2\delta}}{\Delta}g^2R^2(W+\Omega P)^2 + \Delta(R')^2\right], \tag{13}$$

$$\frac{\Delta}{2r^2}(r^3e^{\delta}\Omega')' = 2\kappa^2g^2R^2Pe^{\delta}\frac{W+\Omega P}{r}. \tag{14}$$

We set the metric which approaches asymptotically the BTZ solution when the scalar field falls into a vacuum of the negative value at radial infinity. In this setting, one can find that the cosmological constant C is obtained to be $C = 2(c^4 + b^2 + 2bc^2)$ and as analyzed in [18], the physical parameters (the BTZ mass, the angular momentum and the magnetic flux) is the following:

$$M_{BTZ} = M(r_*) + \frac{r_*^4}{4}e^{2\delta(r_*)}(\Omega'(r_*))^2, \tag{15}$$

$$J = r_*^3e^{\delta(r_*)}(\Omega'(r_*)), \tag{16}$$

$$\Phi = 2\pi P(r_*), \tag{17}$$

where r_* is a sufficiently large value for radial coordinate r.

These equations unfortunately cannot be solved analytically. Then we will find the numerical solution. In the next section, we discuss the relation among the physical parameters for various values of a cosmological constant.

IV. NUMERICAL RESULT AND CONCLUSION

In this paper, we have numerically constructed the non-topological solitions in three-dimensional $U(1)$-gauged $\mathcal{N}=2$ supergravity. The non-topological soliton does not preserve the supersymmetry, in general, because the metric is described by the BTZ solution at radial infinity.

We have found that the region of the solutions can exist for the non-topological solitons in Fig. 1, shown with the BTZ mass M_{BTZ}, the angular momentum J and the magnetic flux Φ for various values of the cosmological constant C. In Fig. 2, we find two branches in a sequence of the solutions which are obtained numerically by the different conditions (values of the scalar field and the vector field at the origin). One branch is the solutions from the origin to the maximal value of the M_{BTZ}, the other is from the point $(0, -1)$ to the maximal value. As studied in [18], one can also compare the results with black hole physics. In the case of the black hole, it is known that a black hole with a larger angular momentum than the critical one has a naked singularity and cannot be the usual black hole. Moreover there is the relation $\sqrt{C}J = M_{BTZ}$ which is satisfied by the extreme black hole. If the non-topological soliton becomes a black hole, the soliton with $\sqrt{C}J < M_{BTZ}$ can become the usual black hole. The non-topological soliton with $\sqrt{C}J > M_{BTZ}$ cannot be the usual black hole and the soliton with a large mass can certainly become an extreme black hole. Thus, the region of the solutions for the non-topological solitons is plausible as self-gravitating systems because all of the maximal values for the soliton's mass exist in the area $\sqrt{C}J < M_{BTZ}$. In Fig 3, if we fit the plotted line with a solid line, one can find that the J is proportional to the square of the Φ. This is the same results as in the Minkowski space-time case, as known in [14–16]. Therefore, the relation between the angular momentum and the magnetic flux is not dependent of the cosmological constant.

There is a phase transition point between the symmetric and the asymmetric phase in the Abelian Chern-Simons Higgs model in Minkowski space-times. On the other hands, in the $U(1)$-gauged $\mathcal{N}=2$ supergravity, differed from the usual phase transition, there is not a

non-topological soliton in the transition point because the metric approaches asymptotically anti de-Sitter space-time. It may be crucial in cosmological situation though this model has still three-dimensions.

In the present paper, we have treated the soliton with zero-winding number case, i.e. $n = 0$ only. We will further investigate the binding energy, the stability of the soliton.

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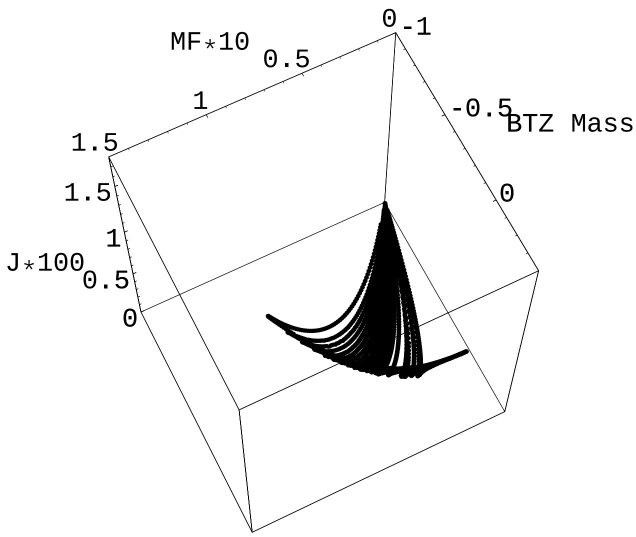

FIG. 1. The region of the non-topological soliton showing with the BTZ mass M_{BTZ}, the angular momentum J, and the magnetic flux Φ for various values of a cosmological constant C.

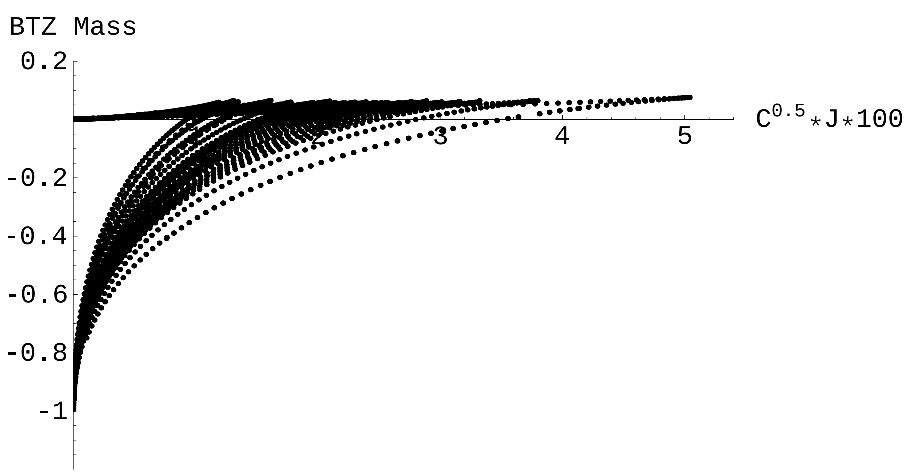

FIG. 2. The region of the non-topological soliton showing with M_{BTZ} and $\sqrt{C}J$

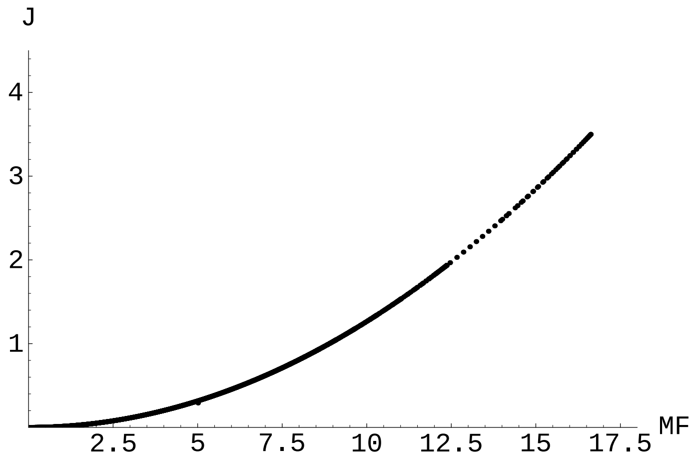

FIG. 3. The region of the non-topological soliton showing with Φ and J

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